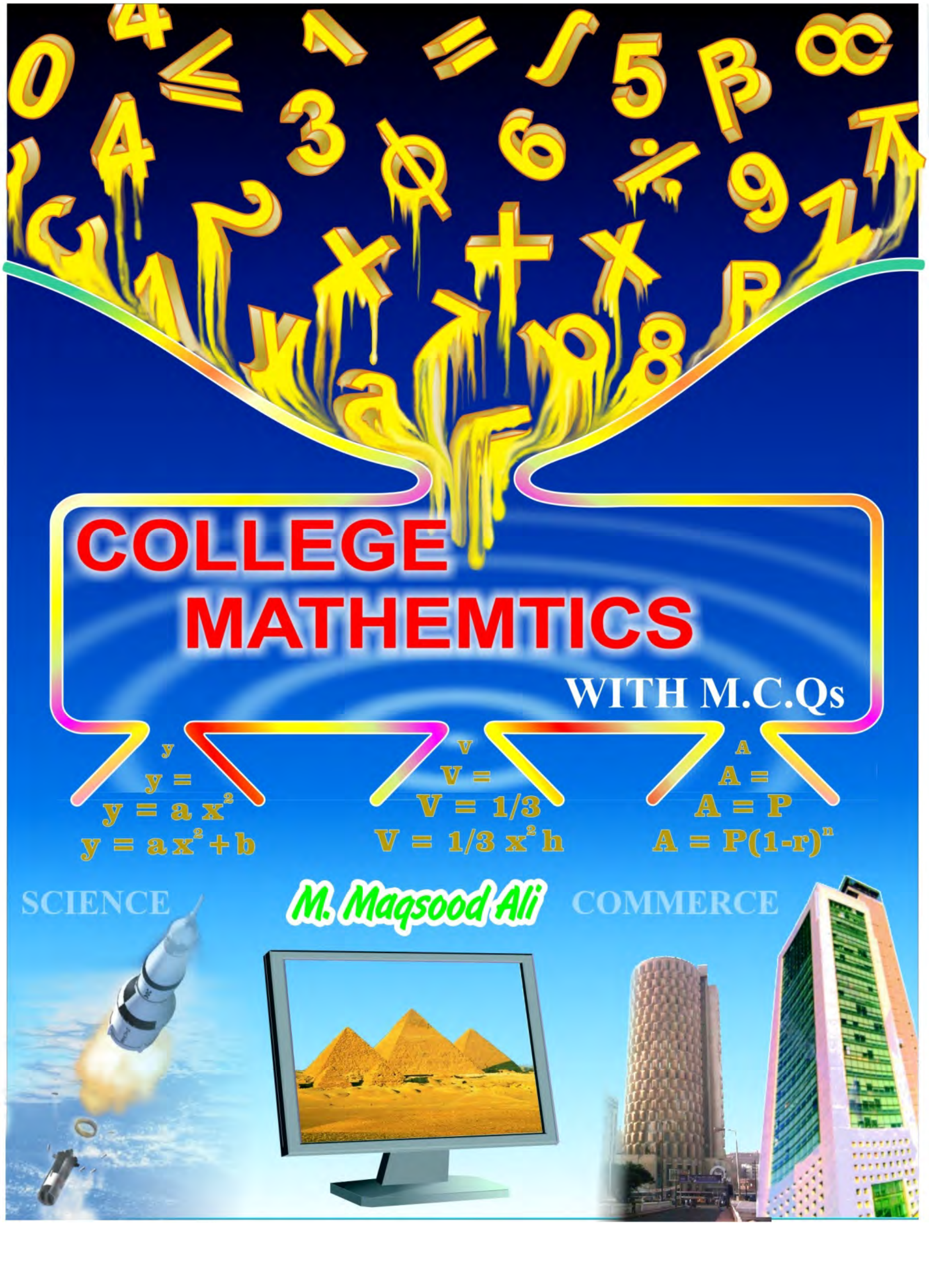
COLLEGE
MATHEMTICS
WITH M.C.Qs
y
y =
$y = a x^2$
$y = ax^2 + b$
V
V =
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$A = P(1-r)^n$
SCIENCE
M. Maqsood Ali
COMMERCE

SECTION A

ARITHMETIC

Chapter 1

PRESENTATION OF NUMBERS

Numbers, letters and symbols are used to express our thoughts in written form. Mathematics is nothing without numbers. The major part of mathematics consist of real numbers. The real numbers are further divided according to the properties of the numbers.

(1) Natural Numbers:

$$1, 2, 3, 4, \ldots$$

(2) Whole Numbers:

$$0, 1, 2, 3, \ldots$$

(3) Integers:

$$0, \pm 1, \pm 2, \pm 3, \ldots$$

(4) Non-negative Integers:

$$0, 1, 2, 3, 4, \ldots$$

(5) Positive Integers:

$$1,2, 3, 4, \ldots$$

(6) Prime Numbers:

$$2, 3, 5, 7, 11, 13, 19, \ldots$$

The prime numbers are the natural numbers which have exactly two factors. One factor is itslef and other is one.

The natural number 1 is not a prime number because it has only one factor.

Note: The three points . . . represent a continuous process follow the pattern.

(7) Rational and Irrational Numbers:

The numbers that can not be written as quotients of two integers are called rational numbers, such as

$$\frac{1}{2}, \frac{7}{3}, \frac{5}{1} \text{ and so on.}$$

The numbers that can not be written as quotients of integers are called irrational numbers, sucah as, $\sqrt{2}$, $\sqrt{5}$, π, e, etc.

The irrati onal numbers of type $\sqrt[n]{a}$ is called surd, such that $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[5]{11}$ etc.

The irrational numbers of type π, e etc. is called transcendental numbers.

(i) *Decimal Representation:*

Both rational and irrational numbers have decimal representation.

Rational numbers can be written down as "finite decimals" or "infinite repeating decimals". For Example

$$\frac{5}{2} = 2.5 \qquad \text{(finite decimals)}$$

$$\frac{14527}{10000} = 1.4527 \quad \text{(finite decimals)}$$

$$\frac{19}{3} = 6.333\ldots \quad \text{(infinite repeating decimals)}$$

$$\frac{23}{27} = 0.851851851\ \ldots \quad \text{(infinite repeating decimals)}$$

$$\frac{335}{111} = 3.0180180180\ \ldots \text{(infinite repeating decimals)}$$

Irrational numbers have non-repeating infinite decimals. For example

$$\sqrt{2} = 141421356\ldots \quad , \quad \sqrt{5} = 2.236067\ldots$$

$$\pi = 3.141592\ldots \quad , \quad e = 2.718281\ldots$$

(ii) *Recurring Decimals:*

The infinite repeating decimals such as 6.333 . . . and 0.851851 851 . . . are called recurring decimals. It can be written as $2.\dot{3}$, $0.\dot{8}\dot{5}\dot{1}$. The dots above the digits indicate that the digit is repeating.

Converting Recurring Decimals into Fractions:

(1) $2.\dot{3} = 2 + \dfrac{3}{9} = \dfrac{7}{3}$

(2) $0.\dot{8}\dot{5}\dot{1} = 0 + \dfrac{851}{999} = \dfrac{23}{27}$

(3) $3.0\dot{1}\dot{8}\dot{0} = 3.0 + 0.0\dot{1}\dot{8}\dot{0} = \dfrac{30}{10} + \dfrac{180}{99 \times 10} = \dfrac{335}{111}$

(4) $3.27\dot{6}\dot{5} = 3.27 + 0.00\dot{6}\dot{5} = \dfrac{327}{100} + \dfrac{65}{99 \times 100} = \dfrac{16219}{4950}$

(8) Real Numbers:

The union of the sets of rational and irrational numbers is the set of real numbers.

DIGIT

A figure (symbol) which make up part of a number is called digit.

(1) There are ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in the ordinary (decimal) number system.

(2) There are two digits 0 and 1 in binary system.

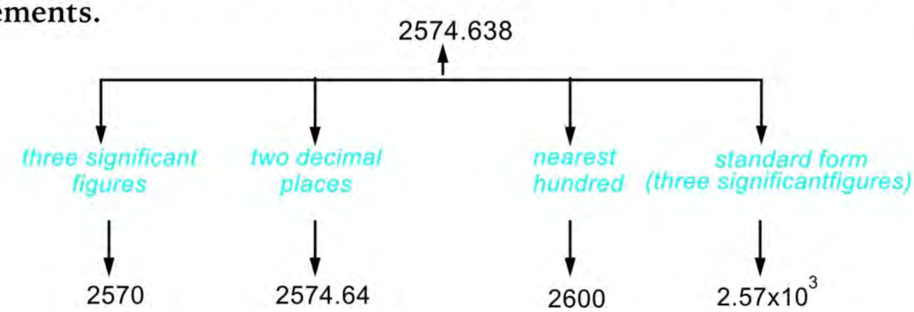
For example: The number 8759 has four digits.

8	7	5	9
↓	↓	↓	↓
thousand digit	*hundred digit*	*ten digit*	*unit digit*

PRESENTATION OF A NUMBER

A number which consists of more than one digit may be rounded of f after a required number of digits.

For example, the number 2574.638 can be written according to the requirements.



(3) NEAREST NUMBER:

The following table help us to understand the position of a digit in a number.

S.No.	Number in 10^n	Name	Prefix
1	10^{15}	Quadrillion	peta
2	10^{12}	Trillion	tera
3	10^{9}	Billion	giga
4	10^{6}	Million	mega
5	10^{3}	Thousand	kilo
6	10^{2}	Hundred	hecto
7	10	Ten	deca
8	$10^{-1}=\frac{1}{10}$	Tenth	deci
9	$10^{-2}=\frac{1}{10^2}$	Hundredth	centi
10	$10^{-3}=\frac{1}{10^3}$	Thousandth	milli
11	$10^{-6}=\frac{1}{10^6}$	Millionth	micro
12	$10^{-9}=\frac{1}{10^9}$	Billionth	nano
13	$10^{-12}=\frac{1}{10^{12}}$	Trillionth	pico
14	$10^{-15}=\frac{1}{10^{15}}$	Quadrillianth	femto

Consider the number 7963.582 and the positions of the digits.

7	9	6	3	.	5	8	2
↓	↓	↓	↓		↓	↓	↓
thousand place	*hundreds place*	*tens place*	*units place*		*tenths place*	*hundredths place*	*thousandths place*

According to the table:

$$1 \text{ Kilogram} = 1000 \text{ grams}$$

$$1 \text{ Kilometre} = 1000 \text{ metres}$$

$$1 \text{ decimetre} = \frac{1}{10} \text{ metres}$$

$$1 \text{ centimetre} = \frac{1}{100} \text{ metres}$$

$$1 \text{ milimetre} = \frac{1}{1000} \text{ metres}$$

For example: The numbers are rounded off as indicated.

(1)	29.63	29.6	to the nearest tenth.
(2)	562.2861	562.29	to the nearest hundredth.
(3)	6832.51	6830	to the nearest ten.
(4)	53917	54000	to the nearest thousand.
(5)	526g	530 g	to the nearest 10 gram.
(6)	26871 km	26900 km	to the nearest 100 kilometre.
(7)	3.692 kg	3.7 kg	to the nearest 0.1 kg.
(8)	5.2831 cm	5.28 cm	to the nearest $^1/_{100}$cm.
(9)	73.632 m	73.6 m	to the nearest $^1/_{10}$m.
(10)	34.28351 m	34.284 m	to the nearest .001 m.

(4) SIGNIFICANT FIGURES:

A digit of a number may or may not significant. To understand it the numbers are divided into two types.

Type 1:

(a) All digits are non-zero.

(b) zeros lie between two non-zero digits.

In both cases (a) and (b) all digits are significant.

For example:

(1)	3258	:	Four significant figures.
(2)	35007	:	Five significant figures.
(3)	305.002	:	Six significant figures.

(2) 0.000592 is a number can be written in standard form as given below.

$$0.000592 = 0.000592 \times 10^{o} = 5.92 \times 10^{-4}$$

Decimal point moves at 4 places to the right (it means the number is multiplied by 10^{4}). The number becomes greater to balance it we subtract 4 from the power of 10^{o}.

For example:

S.No.	Ordinary Number	Standard Form
1	58693	5.8693×10^{4}
2	0.0000863	8.63×10^{-5}
3	852.3	8.523×10^{2}
4	0.063	6.3×10^{-2}

STANDARD FORM OF NUMBERS AND OPERATIONS

The numbers in standard form can be added , subtracted , multiplied and divided according to following rule.

(1) Addition:

$$a \times 10^{m} + b \times 10^{m} = (a + b) \times 10^{m}$$

Power of 10 must be same in each term.

(1) $3.5 \times 10^{5} + 6 \times 10^{4}$

$= 35 \times 10^{4} + 6 \times 10^{4}$

$= (35 + 6) \times 10^{4}$

$= 41 \times 10^{4}$

$= 4.1 \times 10^{5}$

Another Method:

$3.5 \times 10^{5} + 6 \times 10^{4}$

$= 3.5 \times 10^{5} + 0.6 \times 10^{5}$

$= (3.5 + 0.6) \times 10^{5}$

$= 4.1 \times 10^{5}$

(2) Subtraction:

$$a \times 10^{m} - b \times 10^{m} = (a - b) \times 10^{m}$$

Power of 10 must be same in each term.

(2) $8.65 \times 10^{7} - 2 \times 10^{8}$

$= 0.86 \times 10^{8} - 2 \times 10^{8}$

$= (0.86 - 2) \times 10^{8}$

$= -1.14 \times 10^{8}$

(3) $4.5 \times 10^{-3} - 2.3 \times 10^{-4}$

$= 4.5 \times 10^{-3} + 0.23 \times 10^{-3}$

$= (4.5 + 0.23) \times 10^{-3}$

$= 4.27 \times 10^{-3}$

(3) Multiplication:

$$a \times 10^m \times b \times 10^n = ab \times 10^{m+n}$$

(4) $6 \times 10^6 \times 8 \times 10^2$

$= 48 \times 10^8$

$= 4.8 \times 10^9$

(5) $5 \times 10^{-6} \times 6 \times 10^2$

$= 30 \times 10^{-4}$

$= 3 \times 10^{-3}$

(4) Division:

$$a \times 10^m \div b \times 10^n = \frac{a}{b} \times 10^{m-n}$$

(6) $6 \times 10^6 \div 8 \times 10^2$

$= 0.75 \times 10^4$

$= 7.5 \times 10^3$

(7) $5 \times 10^{-6} \div 2 \times 10^{-10}$

$= 2.5 \times 10^4$

EXERCISES A-1

Write the following numbers correct to 3 decimal places.

(1)	35.7856	(2)	873.2754	(3)	8.3948
(4)	32.3896	(5)	105.7996	(6)	24.867321
(7)	101.327592	(8)	204.5684521	(9)	25.869527
(10)	53.799632	(11)	24.99953	(12)	82.93286
(13)	53.245432	(14)	31.98321		

Write the following numbers correct to 3 significant figures.

(15)	8.35928	(16)	82.359	(17)	0.346217
(18)	0.89652	(19)	53.672	(20)	53.927
(21)	81.962	(22)	2.9681	(23)	7.6883
(24)	527.962	(25)	0.003852	(26)	0.082691
(27)	0.00035867	(28)	0.013682	(29)	0.0087232
(30)	0.0009621	(31)	0.096	(32)	0.0087
(33)	0.008	(34)	5.6	(35)	8.2
(36)	0.5	(37)	52	(38)	23
(39)	8	(40)	9	(41)	549687
(42)	268759	(43)	536279	(44)	286215
(45)	738962	(46)	589621	(47)	869521
(48)	599621	(49)	869521	(50)	24568
(51)	859217	(52)	34678.23	(53)	53.682
(54)	486932.3				

Express the following in standard form.

(55)	3890000	(56)	48300000	(57)	5396000
(58)	60000000	(59)	58000000	(60)	6830000
(61)	0.0003	(62)	0.00586	(63)	0.0000089
(64)	0.3	(65)	0.9	(66)	0.86309

Express the following in standard form, correct to two significant figures.

(67)	35896000	(68)	5832000	(69)	285321000
(70)	1059600	(71)	1.962000	(72)	859600
(73)	0.00325	(74)	0.00005	(75)	0.00006
(76)	0.000006859	(77)	0.006993	(78)	0.0038921

(120) The mass of potatoes and tomatoes to the nearest kg are 5 kg and 2 kg respectively. Calculate the least combined mass of potatoes and tomatoes.

EXERCISE A-2

Evaluate the following and express the answer in standard form:

(1) $6 \times 10^3 \times 9 \times 10^4$

(2) $5 \times 10^2 \times 6 \times 10^8$

(3) $9 \times 10^6 \times 6 \times 10^{-3}$

(4) $6 \times 10^{-5} \times 8 \times 10^{-6}$

(5) $3 \times 10^{-2} \times 9 \times 10^3$

(6) $7 \times 10^{-6} \times 9 \times 10^{-4}$

(7) $6.1 \times 10^5 \times 5.8 \times 10^{-9}$

(8) $2.01 \times 10^3 \times 5.1 \times 10^{-8}$

(9) $(8 \times 10^6) \div (4 \times 10^4)$

(10) $(3 \times 10^2) \div (5 \times 10^7)$

(11) $(7 \times 10^{-2}) \div (5 \times 10^{-4})$

(12) $(5 \times 10^4) \div (4 \times 10^8)$

(13) $(2 \times 10^{-3}) \div (5 \times 10^5)$

(14) $(6 \times 10^5) \div (8 \times 10^{-3})$

(15) $(5.1 \times 10^3) \div (1.7 \times 10^8)$

(16) $(2.6 \quad 10^5) \div (5.2 \quad 10^{-8})$

(17) $2 \times 10^3 + 3 \times 10^4$

(18) $5 \times 10^6 + 3 \times 10^5$

(19) $7 \times 10^{-4} + 8 \times 10^{-2}$

(20) $6 \times 10^{-7} + 5 \times 10^{-6}$

(21) $5 \times 10^6 - 7 \times 10^4$

(22) $4 \times 10^3 - 3 \times 10^5$

(23) $3 \times 10^{-5} - 5 \times 10^{-6}$

(24) $8 \times 10^{-7} - 6 \times 10^{-5}$

(25) $5.6 \times 10^8 + 2.6 \times 10^6$

(26) $8.2 \times 10^3 - 9.3 \times 10^4$

(27) $3.2 \times 10^6 - 5.6 \times 10^5$

(28) $6.3 \times 10^8 + 2.5 \times 10^6$

(29) $(3 \times 10^6 \times 5 \times 10^2) + 2 \times 10^7$

(30) $(2 \times 10^2) \div (5 \times 10^{-2}) - 3 \times 10^2$

Evaluate the following, giving your answer in standard form correct to three significant figures.

(31) $5.01 \times 10^6 \times 6.023 \times 10^8$

(32) $2.8 \times 10^3 \times 5.22 \times 10^{-6}$

(33) $3.02 \times 10^{-2} \times 5.1 \times 10^4$

(34) $5.3 \times 10^5 \div 8.5 \times 10^{-5}$

(35) $6.3 \times 10^8 \div 2.7 \times 10^7$

(36) $3.62 \times 10^8 \div 6.349 \times 10^2$

(37) $5.38 \times 10^5 + 6.351 \times 10^4$

(38) $8.932 \times 10^5 + 5.213 \times 10^6$

(39) $9.231 \times 10^5 - 3.287 \times 10^6$

(40) $3.287 \times 10^5 - 7.32 \times 10^3$

M.C.Q's A-1

(1) 25.6952 is rounded off to the nearest hundredth. What is the num ber?
(a) 25.60 (b) 25.69 (c) 24.75 (d) 25.70

(2) 58.65 kg is rounded off to the nearest ten kg. What is the mass in kilograms?
(a) 60 (b) 58.7 (c) 59 (d) 58.6

(3) 3896.568 m is rounded off to the nearest ten metre. What is the length in metres?
(a) 3896.6 (b) 3900 (c) 3890 (d) 3895

(4) 2456.682 g is rounded off to the nearest hundredth of a gram. Wh at is the mass in grams?
(a) 2500 (b) 2400 (c) 2456.68 (d) 2456.60

(5) 48.59 cm is rounded to the nearest cm. What is the length in cm?
(a) 48 (b) 48.5 (c) 48.6 (d) 49

(6) 249.82 grams is rounded off to the nearest gram. What is the mas s in grams?
(a) 249 (b) 249.8 (c) 250 (d) 249.85

(7) 85.692 = _______ correct to one significant figure.
(a) 90 (b) 85.7 (c) 85 (d) 90.7

(8) 76.7996 = _______ correct to five significant figure.
(a) 76.799 (b) 77.000 (c) 76.800 (d) 76.795

(9) 799.96 = _______ correct to one significant figure.
(a) 8 (b) 800 (c) 799 (d) 799.9

(10) 0.005682 = _______ correct to three significant figures.
(a) 0.006 (b) 0.00568 (c) 0.00569 (d) 0.005

(11) 56.892 = _______ correct to the two decimal places.
(a) 56 (b) 56.90 (c) 56.89 (d) 56.85

(12) 5867 =_______ standard form correct to two significant figures.
(a) 5.8×10^3 (b) 5.9×10^{-3} (c) 586×10^1
(d) 5.9×10^3

(13) 8276.27 = _______ standard form correct to three significant fi gures.
(a) 827.6×10^1 (b) 8.27×10^5 (c) 8.28×10^3
(d) 8.267×10^3

(7) $6 \times 10^6 - 3 \times 10^4 =$ _______, standard form.

(a) 6.97×10^6 (b) 3×10^2 (c) 3×10^6

(d) 5.7×10^5

(8) $4 \times 10^{-4} - 7 \times 10^{-5} =$ _______

(a) -3×10^1 (b) 6.6×10^{-4} (c) 3.3×10^{-5}

(d) 3.3×10^{-4}

(9) $0.8 \times 0.6 =$ _______ correct to one significant figures.

(a) 0.4 (b) 1.0 (c) 0.5 (d) None

(10) $1.4 \times 5 =$ _______ correct to two significant figures.

(a) 70 (b) 7.0 (c) 0.7 (d) None

(11) $\dfrac{98.6932}{5} =$ ______ correct to one significan t figure.

(a) 2 (b) 20 (c) 1.9 (d) 19.7

(12) $\dfrac{5869.2}{400} =$ ______ correct to one significan t figure.

(a) 20 (b) 14.7 (c) 1.47×10^1 (d) 10

(13) 24 kg is rounded off to the nearest kg. Which is the correct of the following?

(a) $23.5 \le 24 < 24.5$ (b) $23.5 \le 24 \le 24.5$

(c) $23.5 < 24 \le 24.5$ (d) $23.5 < 24 < 24.5$

(14) 2.5 cm is rounded off to the nearest tenth cm. Which is the correct of the following? (All values are measured in cm).

(a) $2.45 \le 2.5 \le 2.55$ (b) $2.45 < 2.5 \le 2.55$

(c) $2.45 \le 2.5 < 2.55$ (d) $2.45 < 2.5 < 2.55$

(15) 5 g is rounded off to the nearest unit gram. Which of the follow ing is correct?

(a) $4.5 < 5 < 5.5$ (b) $4 \le 5 < 6$

(c) $4.5 \le 5 \le 5.5$ (b) $4.5 \le 5 < 5.5$

(16) The lengths of two sticks are 50 cm and 25 cm both are correct to the nearest cm. What is the smallest combined length in cm of the sticks?

(a) 75.5 (b) 74 (c) 75 (d) 74.5

(17) The mass of sugar is 12.5 kg, correct to the nearest tenth kg and the mass of rice 20 kg nearest to the ten kg. What is the average of the greatest and smallest mass in kg of sugar and rice respectively.

(a) 16.025 (b) 16.25 (c) 14.75 (d) 16.475

Chapter 2

FRACTION S , MULTIPLES AND FACTORS

FRACTIONS

a, b and c are integers where $b \neq 0$.

(1) $\frac{a}{b}$ is a proper fraction if $b > a$.

For example $\frac{5}{8}, \frac{3}{10}, \frac{19}{25}$ are proper fractions.

(2) $\frac{a}{b}$ is an improper fraction if $b < a$.

For example $\frac{5}{2}, \frac{8}{3}, \frac{25}{19}$ are improper fractions.

(3) $c\frac{a}{b}$ is a mixed number where $b < a$.

$c\frac{a}{b}$ is a short form of $\left(c + \frac{a}{b}\right)$.

For example $5\frac{3}{6}, 2\frac{1}{5}, 3\frac{9}{16}$ are mixed numbers.

EXERCISE A-3

Express the following improper fractions as a mixed number.

(1) $\frac{43}{5}$. (2) $\frac{84}{13}$ (3) $\frac{93}{2}$

(4) $\frac{105}{8}$ (5) $\frac{105}{25}$ (6) $\frac{520}{6}$

Express each mixed number as a improper fraction.

(7) $15\frac{2}{3}$ (8) $4\frac{1}{2}$ (9) $50\frac{3}{5}$

(10) $103\frac{3}{4}$ (11) $55\frac{2}{7}$ (12) $83\frac{2}{5}$

Express the following as fractions in their lowest terms:

(13) 8.35 (14) 3.6386 (15) 55.55

(16) 4.821 (17) 5.435 (18) 3.8

Express each of the following as a decimal correct to four significant figures.

(19) $\frac{33}{7}$ (20) $\frac{112}{6}$ (21) $\frac{57}{7}$

(22) $\frac{33}{58}$ (23) $\frac{1}{100}$ (24) $\frac{52}{7}$

Express each of the following as a recurring decimal.

(25) $\frac{10}{3}$ (26) $\frac{52}{9}$ (27) $\frac{13}{99}$

(28) $\frac{508}{99}$ (29) $\frac{208}{33}$ (30) $\frac{265}{11}$

(31) $\frac{2081}{99}$ (32) $\frac{2518}{99}$ (33) $\frac{194}{37}$

DIVISOR , MULTIPLE AND FACTOR

For Universal Set $\mathbb{Z}$:

If $\mathbb{Z}$ (set of integers) is the universal set and a, b, ab, $b/_a$ are the elements of $\mathbb{Z}$, the multiple and factor can be explained as

(1) Divisor:

An integer "a" is a divisor of another integer "b" if b is exactly divisible by a.

For example: (1) 3 is a divisor of 6.

(2) 1, 2, 3, 6 are divisors of 6.

(2) Multiple:

An integer b is multiple of an integer a if "b is exactly divisible by a".

For example: (1) 6 is multiple of 3.

(2) 6 is multiple of 1, 2, 3, 6.

(3) The multiples of 2 are 2, 4, 6, 8, . . .

(4) The multiples of 5 are 5, 10, 15, 20, . . .

(5) The multiples of 7 are 7, 14, 21, 28, . . .

(3) Factor:

An integer a is factor of another integer b if "b is exactly divisible by a".

For example: (1) 3 is a factor of 6.

(2) 1, 2, 3, 6 are factors of 6.

(4) Prime Factor:

A factor is said to be a prime factor if it is a prime number.

MULTIPLE AND FACOTR

Multiple and factor can be explained by a sentence.

$$\left.\begin{array}{l} b \text{ is multiple of } a \\ a \text{ is factor of } b \end{array}\right\} \text{if "b is exactly divisible by a"}$$

where a and b are integers.

For Universal Set $\mathbb{R}$:

$\mathbb{R}$ (set of real numbers) is the universal set.

If a and b are real numbers, then ab is also is real number. The numbers a and b are called algebraic factors of ab.

Example 1: Find all prime factors of 210.

Solution:-

The prime factors of 210 are,

2	210
5	105
3	21
7	7
	1

2, 3, 5 and 7.

Example 2: Find all multiples of 5 between 8 and 42.

Solution:-

All multiples of 5 between 8 and 42 are 10, 15, 20, 25, 30, 35 and 40.

H.C.F , L.C.M AND L.C.D

(1) H.C.F. (Highest Common Factor):

A greatest number which is a factor of two or more numbers is said to be H.C.F. of these numbers.

Explanation:

The common factors and the highest common factor of 42 and 70 step by step.

Step 1: All factors of 42:
1, 2, 3, 6, 7, 14, 21 and 42.

Step 2: All factors of 70:
1, 2, 5, 7, 10, 14, 35 and 70.

Step 3: The "common factors" of 42 and 70:
1, 2, 7, 14.

Step 4: The "highest common factor" of 42 and 70:
14.

Another Method: (Short Cut)

$$\begin{array}{c|c} 2 & 42\ ,\ 70 \\ \hline 7 & 21\ ,\ 35 \\ \hline & 3\ ,\ 5 \end{array}$$

$\therefore$ H.C.F. $= 2 \times 7 = 14$

(2) L.C.M. (Lowest Common Multiple):

A least number which is a multiple of two or more numbers is said to be L.C.M. of three numbers.

Explanation:

The common multiples and lowest commn multiple of 4 and 6 step by step.

Step 1: All multiples of 4:
4, 8, 12, 16, 20, 24, 28, 32, 36, 40, . . .

Step 2: All multiples of 6:
6, 12, 18, 24, 30, 36, 42, . . .

Step 3: Common multiples of 4 and 6:
12, 24, 36, . . .

Step 4: Lowest common multiple of 4 and 6:
12

Another Method: (Short Cut)

$$\begin{array}{c|c} 2 & 4\ ,\ 6 \\ \hline & 2\ ,\ 3 \end{array}$$

$\therefore \quad$ L.C.M. $= 2 \times 2 \times 3 = 12$

(3) L.C.D. (Lowest Common Denominator):

The lowest common multiple (L.C.M.) of the denominators of two or more fractions is said to be L.C.D.

Exaplantion:

$\frac{2}{5}, \frac{3}{10}$ and $\frac{1}{3}$ are three fractions.

$$\begin{array}{c|ccccc} 5 & 5 & , & 10 & , & 3 \\ \hline & 1 & , & 2 & , & 3 \end{array}$$

$\therefore \quad$ L.C.D. $= 2 \times 3 \times 5 = 30.$

Example 3: Find H.C.F. and L.C.M. of $3^3 \times 5^2 \times 7$ and $3^2 \times 5^3 \times 2$.

Solution:-

$$\text{H.C.F.} = 3^2 \times 5^2 = 225$$
$$\text{L.C.M.} = 3^3 \times 5^3 \times 7 \times 2 = 47250$$

Relation between L.C.M. and H.C.F.:

The product of H.C.F. and L.C.M. of two numbers is equal to the product of the numbers.

Explanation:

10 and 15 are two integers.

L.C.M. of 10 and 15 = 30

H.C.F. of 10 and 15 = 5

$$\text{Product of H.C.F. and L.C.M.} = 5 \times 30 = 150$$
$$\text{Product of the numbers 10 and 15} = 10 \times 15 = 150$$

EXERCISE A-4

Find all prime factors of the following:

(1) 30 (2) 36 (3) 74

(4) Write all multiples of 3 between 5 and 20.

(5) Write all common factors of 30 and 80.

(6) Write all common multiples of 3 and 4 between 4 and 35.

Find H.C.F. and L.C.M. of the following:

(7) 15, 10, 25 (8) 18, 36, 12 (9) 14, 42, 6

Find H.C.F. and L.C.M. of the following:

(10) $2^3 \times 3^2 \times 5$ and $2^2 \times 3^3 \times 7$ (11) $2^2 \times 3^2$ and $5^2 \times 2$

(12) $2^3 \times 3$ and $2^2 \times 3^2$

(13) $3^2 \times 7 \times 5^3$ and $2 \times 3^3 \times 5^2$

(14) Find the product of two numbers. The H.C.F. and L.C.M. of the numbers are 12 and 36 respectively.

(15) The H.C.F. and L.C.M. of two numbers are 6 and 60 respectively. Find the other number if one of the number is 12.

M.C.Q's A-3

(1) What is the lowest common factor of the integers x and y, such that

$$x = 5^2 \times 3^3 \times 2 \quad \text{and} \quad y = 5 \times 3^2 \times 4$$

(a) 90 (b) 2700 (c) 5400 (d) None

(2) What is the highest common multiple of the integers x and y, such that

$$x = 2^4 \times 3^2 \quad \text{and} \quad y = 3^3 \times 4 \times 5$$

(a) 180 (b) 2160 (c) 36 (d) 9

(3) The product of two integers is 960. The L.C.M. of the numbers is 120.What is the H.C.F.?

(a) 16 (b) 24 (c) 72 (d) 8

(4) The L.C.M. and H.C.F. of two numbers are 144 and 4 respectively. What is the product of the numbers?

(a) 576 (b) 1152 (c) 432 (d) 1728

(5) Ali, Babar and Sabir always sit on a particular table to take tea in a hotel. Ali, Babar and Sabir take tea after every 30, 40 and 60 minutes respectively. They take tea at 9:00 a.m. together. At what time they take tea again together.

(a) 12:00 a.m. (b) 9:00 p.m. (c) 10:30 a.m.

(d) 11:00 a.m.

(6) There are four signals on a straight road. The four signals will be red after every 40, 50, 50 and 60 seconds respectively. The four signals become red simultaneously at 5:00 p.m. After how many minutes they will be red togather again?

(a) $3\frac{1}{3}$ (b) $8\frac{1}{2}$ (c) 10 (d) $2\frac{1}{2}$

(7) 35 and 25 are two integers. What is the smallest integer multiple of both integers.

(a) 5 (b) 175 (c) 875 (d) 60

(8) 10, 15 and 20 are the three integers. What is the greatest integer factor of all three integers.

(a) 300 (b) 30 (c) 60 (d) 10

(9) What is the common multiple of 12 and 8?

(a) 6 (b) 48 (c) 4 (d) 18

(10) What is the common factor of 6 and 9?

(a) 9 (b) 54 (c) 18 (d) 3

PROPERTIES:

(1) $\sqrt[m]{ab} = \sqrt[m]{a}\,.\,\sqrt[m]{b}$ (2) $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$

(3) $\sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n$ (4) $\sqrt[m]{\sqrt[n]{b}} = \sqrt[mn]{b}$

(5) If m is odd $\sqrt[m]{-b} = -\sqrt[m]{b}$

Example 5: Simplify:

(1) $\sqrt[3]{40} + \sqrt[3]{135} - \sqrt[3]{-5}$ ***(2)*** $\sqrt[5]{8}\,.\,\sqrt[5]{4}$

Solution:-

(1) $\sqrt[3]{40} + \sqrt[3]{135} - \sqrt[3]{-5} = 2\sqrt[3]{5} + 3\sqrt[3]{5} + \sqrt[3]{5} = 8\sqrt[3]{5}$

(2) $\sqrt[5]{8}\,.\,\sqrt[5]{4} = \sqrt[5]{8 \times 4} = \sqrt[5]{32} = 2$

RADICALS AND INDICES

Radical form $\longleftarrow$ $\sqrt[3]{5} = 5^{1/3}$ $\longrightarrow$ *index form.*

Index:

m is the index in the following:

$$x^m \quad , \quad \sqrt[m]{b} \quad , \quad m = \log p$$

Example 6: Express the following in index form:

(1) $\sqrt[3]{6}$ (2) $\sqrt[5]{3^2}$ (3) $\sqrt{5^3}$

Solution:-

(1) $6^{1/3}$ (2) $3^{2/5}$ (3) $5^{3/2}$

Example 7: Express the following in radical form:

(1) $5^{1/3}$ (2) $3^{2/3}$ (3) $6^{0.2}$ (4) $3^{-1/5}$

Solution:-

(1) $5^{1/3} = \sqrt[3]{5}$ (2) $3^{2/3} = \sqrt[3]{3^2} = \sqrt[3]{9}$

(3) $6^{0.2} = 6^{1/5} = \sqrt[5]{6}$ (4) $3^{-1/5} = \dfrac{1}{3^{1/5}} = \dfrac{1}{\sqrt[5]{3}}$

Without using calculator find the value of:

(43) $\sqrt{3579}$ if $\sqrt{3.579} = 1.89$ and $\sqrt{35.79} = 5.98$

(44) $\sqrt{0.0003869}$ if $\sqrt{386.9} = 19.7$ and $\sqrt{38.69} = 6.22$

(45) $\sqrt{656312}$ if $\sqrt{656.312} = 25.62$ and $\sqrt{6563.12} = 81.013$

(46) $\sqrt{0.00597}$ if $\sqrt{5.97} = 2.44$ and $\sqrt{59.7} = 7.73$

Simplify the following:

(47) $5\sqrt[3]{2} - 3\sqrt[3]{2} + 7\sqrt[3]{-2}$ (48) $5\sqrt[3]{16} + 9\sqrt[3]{54}$

(49) $2\sqrt[5]{-6} - 9\sqrt[5]{6} + 10\sqrt[5]{6}$ (50) $\sqrt[3]{-16} + 5\sqrt[3]{54}$

(51) $2\sqrt[3]{\frac{-5}{8}} + 3\sqrt[3]{\frac{5}{27}}$ (52) $5\sqrt{\frac{27}{49}} - 7\sqrt{3}$

(53) $5\sqrt{7} + 3\sqrt{7} - 8\sqrt{3}$ (54) $\sqrt[3]{8} + \sqrt[5]{(32)^2}$

(55) $\sqrt{(49)^3} - \sqrt[3]{(64)^2}$ (56) $\sqrt[5]{(-32)^3} + \sqrt{(100)^5}$

Simplify the following:

(57) $\sqrt{3}\,.\,\sqrt{12}$ (58) $\sqrt[3]{10}\,.\,\sqrt[3]{100}$

(59) $\dfrac{\sqrt[3]{48}}{\sqrt[3]{6}}$ (60) $\sqrt[5]{8}\,.\,\sqrt[5]{-8}$

Simplify the following:

(61) $16^{3/2} + 125^{2/3}$ (62) $2^{1/3}\,.\,2^{2/3} + 25^{1/2}$

(63) $5^{2/3} \div 5^{5/3}$ (64) $2^{5/2}\,.\,2^{3/2} - 2^{1/2} \div 2^{3/2}$

(65) $\left(\frac{5}{3}\right)^{o} \times \left(\frac{1}{8}\right)^{3} \div \left(\frac{3}{2}\right)^{2}$ (66) $5^3 + 5^2 + 5^1$

(67) $3^2 + 2^1 - 3^0$ (68) $5^2\,.\,5^3 + 5^4 \div 5^2$

(69) $5^2 \times 5^3 \div 5^4$ (70) $6^{-3} \div 6^2 \times 6^5$

(71) $2^5 + 2^3 \div 2^5 \times 2^{-3}$ (72) $(-3)^3 \div (-3)^2 \times (-3)^1 + (-3)^2$

(73) $\left(\frac{1}{2}\right)^{0} \times \left(\frac{1}{2}\right)^{2} \div \left(\frac{1}{2}\right)^{3}$ (74) $\left(\frac{-3}{2}\right)^{3} + \left(\frac{-3}{2}\right)^{2} \div \left(\frac{9}{16}\right)^{1/2}$

(75) $8^2 \times 3^3 \div 2^2 \times 5^{-2}$ (76) $8^{2/3} \div \sqrt[3]{27}$

(77) $\sqrt[5]{64} \div 8^{2/3} \times \sqrt[3]{64}$ (78) $\sqrt[3]{216} \times 49^{3/2} \div 7$

(79) $\sqrt[3]{-54}\,/\sqrt[3]{2}$ (80) $\sqrt[3]{-8}\,/\sqrt[5]{-32}$

M.C.Q's A-4

(1) $\sqrt[3]{-8} = ?$

(a) -2 (b) $2i$ (c) 2 (d) -2.8

(2) $\sqrt{-16} = ?$

(a) -4 (b) 4 (c) $4i$ (d) None

(3) $\sqrt[3]{2^2} = ?$

(a) $2^{3/2}$ (b) $4^{1/3}$ (c) 8 (d) 2^6

(4) $\sqrt[3]{-2^2} = ?$

(a) $4^{1/3}$ (b) -64 (c) $1/64$ (d) None

(5) Which of the following is the surd?

(a) $\sqrt{16}$ (b) $\sqrt{-6}$ (c) $\sqrt{8}$ (d) $\sqrt[3]{27}$

(6) Which of the following is the surd?

(a) $\sqrt[3]{8}$ (b) $\sqrt[3]{4}$ (c) $5\sqrt{25}$ (d) $5\sqrt{-3}$

(7) Which of the following are similar radicals?

(a) $\sqrt{8}, \sqrt{16}$ (b) $\sqrt[3]{6}, \sqrt{6}$ (c) $5\sqrt[3]{2}, \sqrt[3]{16}$

(d) $\sqrt{-4}, \sqrt{4}$

(8) Which of the following are similar radicals?

(a) $\sqrt[3]{8}, \sqrt{8}$ (b) $\sqrt[5]{3}, \sqrt[5]{6}$ (c) $2\sqrt{5}, \sqrt{10}$

(d) $5\sqrt{12}, \sqrt{3}$

(9) $\sqrt{6} + \sqrt{10} = ?$

(a) 8 (b) 4 (c) 4.5 (d) None

(10) $\sqrt[3]{2} \cdot \sqrt[3]{4} = ?$

(a) $\sqrt[9]{8}$ (b) 2.5 (c) 2 (d) None

(11) $\sqrt{90000} = ?$

(a) 3000 (b) 300 (c) 30 (d) 30000

(12) $\sqrt{0.0064} = ?$

(a) 0.8 (b) 0.008 (c) 0.08 (d) 0.0008

(13) $\sqrt{14.4 \times 10^{-3}} = ?$

(a) 1.2 (b) 0.12 (c) 12 (d) 0.012

(14) $\sqrt[3]{0.27 \times 10^{-4}} = ?$

(a) 0.03 (b) 0.3 (c) 0.003 (d) 0.0003

(15) $\sqrt[3]{12.5 \times 10^{4}} = ?$

(a) 250 (b) 500 (c) 5000 (d) 50

(16) $\sqrt{2437} = ?$ if $\sqrt{2.437} = 1.56$ and $\sqrt{24.37} = 4.94$.

(a) 494 (b) 156 (c) 49.4 (d) 15.6

(17) $\sqrt[3]{4863} = ?$ if $\sqrt[3]{4.863} = 1.69$ and $\sqrt[3]{486.3} = 7.86$.

(a) 786 (b) 16.9 (c) 78.6 (d) 169

(18) $\sqrt[3]{-27} + 5^{o} - 2\sqrt{4} = ?$

(a) –6 (b) –3 + 3i (c) 12 (d) None

(19) $\sqrt[5]{(-32)^3} + \sqrt[3]{-3} \,.\, \sqrt[3]{9}$

(a) –15 (b) 8i – 3 (c) 8 + 3i (d) –11

(20) $\sqrt{(-4)^3} + 6\sqrt{4} = ?$

(a) 4 (b) 12 + 8i (c) 12 – 2i (d) None

(21) $4^{3/2} - 27^{2/3} = ?$

(a) 6 (b) –1 (c) 10 (d) –5

(22) $4^2 - 3^{o} \times \left(\frac{1}{2}\right)^{-3} = ?$

(a) 120 (b) 16 (c) –8 (d) 8

(23) $3^{1/3} \div 3^{4/3} = ?$

(a) 3 (b) 1/3 (c) 9 (d) 27

(24) $\sqrt[3]{-2} \,.\, (4)^{1/3}$

(a) 8 (b) 2i (c) –2 (d) None

(25) $\sqrt{2} \,.\, 6^{1/2} \,.\, \left(\frac{1}{3}\right)^{-1/2}$

(a) 4.32 (b) $\sqrt{4}$ (c) $\sqrt{3}$ (d) 6

(10) $1\ m^2 = 1000\ mm \times 1000\ mm = 10^6\ m^2$

SCALES AND MAPS

You cannot draw the actual figure of a house with its actual measurment on a page. It is only possible when you suppose a scale.

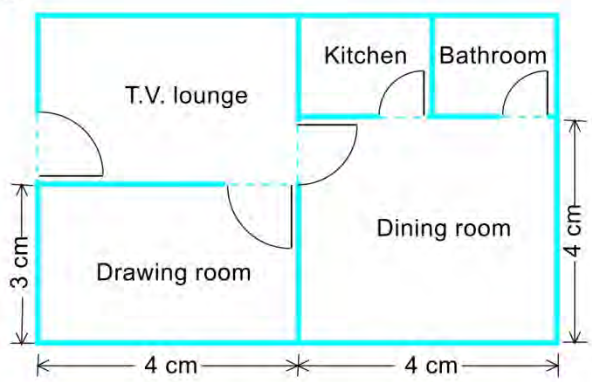
For example, there is a rectangular field of length 800 m and width 600 m. The map of this rectangular field can be drawn using a scale 100 m = 1 cm.

Example 1: A map of a house is given.

The map is drawn using a scale 6 ft (actual) = 1 cm (on map). Find

(i) Actual length and width of drawing room.

(ii) area of dining room in square yards.



Solution:-

(i) 1 cm (map) = 6 ft (actual)

$\Rightarrow$ 4 cm (map) = 24 ft (actual) and 3 cm (map) = 18 ft (actual)

$\therefore$ Length = 24 ft and width = 18 ft

(ii) Area = 4 cm × 4 cm = 16 cm^2

$\because$ 1 cm (map) = 6 ft (actual)

$\Rightarrow$ 1 cm (map) = 2 yards (actual)

$\because$ 1 cm^2 (map) = 4 $yards^2$ (actual)

$\Rightarrow$ 16cm^2 (map) = 64 $yards^2$ (actual)

Area of dining room = 64 square yards.

(b) $1 : 500 \Rightarrow 1\text{cm} = 500 \text{ cm}$

$$\Rightarrow \quad 1 \text{ cm} = 5\text{m} \Rightarrow 1 \text{ cm}^2 = 25\text{m}^2$$

$$200 \text{ cm}^2 = 5000 \text{ m}^2$$

The actual area of the ground is 5000 m^2.

EXERCISE A-6

(1) A length 2 cm on a map represents an actual distance 4 m. Calculate
   (i) the length of a wall of a house 20 m long on the map.
   (ii) the actual area of the garden which is 70 cm^2 on the map.

(2) A map of a park is drawn to a scale of 4 cm^2 (map) = 9 m^2 (actual). Calculate
   (i) the perimeter of the park on the map if the actual perimeter is 300 m.
   (ii) the actual area of a part of the park which is 80 cm^2 on the map.

(3) A model of a ship is made to a scale 1 m^3 (model) = 10^6 m^3 (actual). Calculate
   (i) the height of the pole on the model which is 200m high.
   (ii) the actual area of a side which is $1/2$ m^2.

(4) A map is drawn to a scale of 1 : 10000. Calculate
   (i) the actual length of a road which is 5cm on the map.
   (ii) the actual area which is100 cm^2 on the map.
   (iii) the length on the map of a wall whose actual length is 50 km.
   (iv) the area of a park on the map whose actual area is 100 km^2.
   (v) the area and length on the map if actual area and length are 600 m^2 and 500 m respectively.

(5) A map is drawn to a scale of 1 cm (map) = 5 km (actual). Calculate
   (i) the actual length which is 10 cm on the map.
   (ii) the actual area which is 64 cm^2 on the map.
   (iii) the length on the map which is 40 km actually.
   (iv) the area on the map which is 400 km^2
   (v) the length on the map while actual length is 5000 m.
   (vi) the area on the map while the actual area is 250000m^2.

M.C.Q's A-5

(1) A map is drawn to scale of 1 : 50. The length of a side of a room is 4 m. What is the length, in cm, on the map?

(a) 20 (b) 4 (c) 8 (d) 16

(2) A map is drawn to a scale of 2 cm (map) = 5 m (actual). The actual length of a straight road is 120 m. What is length, in cm, of the road on the map?

(a) 60 (b) 24 (c) 17.14 (d) 48

(3) A map is drawn to a scale of 2 cm (map) = 5 m (actual). The area of a garden is 2500 m^2. What is its area, in cm^2, on the map?

(a) 1000 (b) 400 (c) 100 (d) 357

(4) A map is drawn to a scale of 1 : 50. The area of a park is 2500 m^2. What is the area, in cm^2, of the park on the map?

(a) 10000 (b) 5000 (c) 12.5 (d) None

(5) A map is drawn to a scale of 4 cm^2 (map) = 9m^2 (actual). The length of a wall is 9 m. What is the length, in cm, of this wall on the map?

(a) 4 (b) 20.25 (c) 25 cm (d) 6 cm

(6) A map is drawn to a scale of 4 cm^2 (map) = 9 m^2 (actual). The length of a rectangular plot on the map is 18 cm. What is the actual length in m?

(a) 3.6 (b) 8 (c) 27 (d) 40.5

(7) A map is drawn to a scale of 1 : 5000. The actual area of a park is 2500 m^2. What is the area, in cm^2, on the map?

(a) 100 (b) 500 (c) 12.5 (d) 250

(8) A map is drawn to a scale 1 : 5000. The area on the map of a plot is 200 cm^2. What is the actual area in m^2?

(a) 40000 (b) 40 (c) 25000 (d) 5000

(9) A model of a house is made to a scale of 1 : 10. The same material (cement+send) is used to construct the house and model. If 5 kg cement is used to build the model. How many bags of the cement will be used to construct the house; (50 kg = 1 bag of cement).

(a) 1 (b) 100 (c) 50 (d) 25

(10) A model is made of a scale of 1 : 10. The model is painted in 2 litres distember. How many litres distember will be used on the house if the thickness of the colour are same on the model and the house.

(a) 40 (b) 20 (c) 200 (d) 100

(11) A model of a machine is made to a scale of 1 : 20. If 5 kg iron is used to make the model. How many kilograms of iron will be used to make the machine?

(a) 40000 (b) 2000 (c) 100 (d) None

(12) A model of a tank is made to a scale 1 : 10. The maximum volume the tank holds is 8000 litres. How many litres can be held by the model?

(a) 1.25 (b) 80 (c) 800 (d) 8

(13) The model of a boat is made to a scale of 1 : 5. The model boat can swim with maximum mass of 6 kg. How much mass, in kg, can be loaded on the original boat?

(a) 300 (b) 30 (c) 750 (d) 1200